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ADDITIVE INDEX ANALYSIS APPROACH

This is a new approach to carry out an additive index factor analysis. The novelty is, that during the decomposition of the total growth of the resulting quantity into the corresponding specific subgrowths, the structural characteristics of the extensive factor is *clearly* identified. The scheme of the analysis is presented closer to the reality, the factor subgrowths are specified much more easily and simply, at a much clearer formal external entry.

The problems arising as a result of the decomposition of the total growth have been discussed for both types of weights – basic weight and survey weight. The differences between both types of weighting are pointed out, as well as the interrelations between them. The derived theoretical formulations are supported by the elaborated real example from the Bulgarian demographic practice.

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The article discusses a specific issue of the index analysis. The discussed idea focuses a more different approach in the course of the additive index factor analysis. Actually the case when the aggregate is uniform, and the resulting quantity is presented as a sum of the products of the factor quantities.[1]

The highlight here is, that at the very decomposition of the total growth of the resulting quantity to identify *transparently* the structural quantities of the extensive factor as well. The immediate result thereof is a simplified detailed description of each sub-growth, in view of increasing the number of components of the total growth and their simplified entry.[2] It means more opportunities for the analysis both from the point of view of the detailed limiting of growths, and the identification of their content as far as the sources are concerned and the direction of their factor impact.

It is well known, that the analysis could be made from two starting points: *at the basic weight and at the survey weight.*

Considering this case we deal with a demographic statistics issue. Let us assume that the object of this study is the increased number of deaths in Bulgaria between two consecutive periods – basic period *(2000)* and survey period *(2000).* The total number of deaths during the basic period is indicated with M_0, and during the survey period - with M_1. We assume also, that this number has been considered and analyzed in view of the key demographic indicators – the age. It is specified by means of x = 0,1,2,..., ω, where ω is the utmost age limit.

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[1]The idea for this approach dates back to the year 1981 in view of the scientific study managed by the author on the vital potential of the inhabitants in Bulgaria (See Life potential of the population of PR Bulgaria. Sofia, 1982).

[2] Prof. N. Velichkova was also concerned with the necessity of increasing the number of factor sub-growths. The methodological approaches of the two authors are different. See *Velichkova, N.* Statistical methods for studying and forecasting the development of socio-economic phenomena. Sofia, 1981, p. 9. Some aspects of this issue are also considered by *Gatev K.* Introduction into statistics. Sofia.,1995, p. 370.

In this case the total number of deaths is the sum of the deaths in different ages, i.e. there are equations $M_0 = \sum_{x=0}^{\omega} M_{0,x}$ during the basic period and $M_1 = \sum_{x=0}^{\omega} M_{1,x}$ during the survey period.

We also consider, that the number of the deaths at a given age depends both on the age limit of mortality, and on the number of the population at this age. We start from the fact, that the mortality at a given age of x years (m_x) is defined as a ratio of the density of the deaths at that age (M_x)[3] to the mean quantity of the inhabitants at the same age - $\overline{S}_x$. Or $m_x = \frac{M_x}{\overline{S}_x}$, where we find, that $M_x = m_x \overline{S}_x$. Consequently we could note down the densities as suitable for the analysis:

$$M_0 = \sum_{x=0}^{\omega} M_{0,x} = \sum_{x=0}^{\omega} m_{0,x} \overline{S}_{0,x} \text{, resp. } M_1 = \sum_{x=0}^{\omega} M_{1,x} = \sum_{x=0}^{\omega} m_{1,x} \overline{S}_{1,x}.$$ [4]

Therefore we come to the above-mentioned form of the relation between the resulting and factor quantities – a sum of the products of the age limits of mortality (intensive factor) and the corresponding mean inhabitants by age (extensive factor).

The index theory decomposes the total growth according to one of the two alternative schemes:

$$M_1 - M_0 = \sum M_1 - \sum M_0 = \sum m_1 S_1 - \sum m_0 S_0 = \sum (m_1 - m_0) S_1 + \sum (S_1 - S_0) m_0 =$$
$$= \sum (m_1 - m_0) S_0 + \sum (S_1 - S_0) m_1$$

It is assumed, that each of the two subgrowths reflects separately the influence from the changes of ASMR (age-specific mortality rate) and the changes in the number of inhabitants by age, respectively at basic and survey weights.

In order to avoid the inconvenience from using weights from different periods, a third subgrowth should be included. Hence the mean mortality is introduced:

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[3] By means of density the events are carried for one calendar year. If in such case $M_0'(t)dt$ is the increase in the integral function of the deaths during the elementary time interval dt, measured in years, the density of the deaths for this period is $M_0'(t)$ - the derivative of the function or the differential function of the deaths. Therefore the periods considered here are one-year periods with available equality between the density of the deaths an their number during the period.

[4] Further the limits of the sums, the age indications and the sign for mean indication of the inhabitants – the line above the symbol $\overline{S}$ shall be omitted for convenience.

$$M_1 - M_0 = \sum m_1 S_1 - \sum m_0 S_0 = \overline{m}_1 \sum S_1 - \overline{m}_0 \sum S_0 ,$$

where $\overline{m}_0$, resp. $\overline{m}_1$ are respectively the total (mean) mortality of the population during the basic period and the survey period.

We shall begin with the analysis at basic weights.

1. Additive analysis at basic weights

This decomposition scheme functions only with basic weights, and therefore $\overline{m}_1 = \overline{m}_0 + \Delta_{\overline{m}}$, a $S_1 = S_0 + \Delta_s$ and we replace:

$$M_1 - M_0 = \left(\overline{m}_0 + \Delta_{\overline{m}}\right)\sum\left(S_0 + \Delta_s\right) - \overline{m}_0 \sum S_0 = \overline{m}_0 \sum \Delta_s + \Delta_{\overline{m}} \sum S_0 + \Delta_{\overline{m}} \sum \Delta_s =$$
$$= \overline{m}_0 \sum\left(S_1 - S_0\right) + \left(\overline{m}_1 - \overline{m}_0\right)\sum S_0 + \left(\overline{m}_1 - \overline{m}_0\right)\sum\left(S_1 - S_0\right).$$

The usual interpretation of the three subgrowths is as follows:

a) The first component $\overline{m}_0 \sum\left(S_1 - S_0\right)$ is accepted as an evaluation of the share of the total increase in the deaths, resulting *only* from the changes in the number of inhabitants. The assessment however is made at the level of the basic mean mortality $\overline{m}_0$, which on its part is determined not only by the ASMR, but also by the structure of the population during that period. It means, that the structure is implicitly included in that component. As stipulated in the new approach its involvement should become open. Consequently the modification is as follows:

$$\overline{m}_0 \sum\left(S_1 - S_0\right) = \left(\sum S_1 - S_0\right)\sum m_0 s_0 = I_0 ,$$ [5]

where $s_0 = \dfrac{S_0}{\sum S_0}$, a $\sum s_0 = 1.$

The quantities s_0 refer to each age $x\left(x = 0,1,2,...,\omega\right)$ and determine the age structure of the population during the basic period. This is the structural characteristics the extensive factor, introduced additionally and obviously as stipulated in the new approach.

b) The second component $\left(\overline{m}_1 - \overline{m}_0\right)\sum S_0$ is assumed in general as a share of the total growth, due to the changes in the average level of the intensive factor. Actually three types of influences are included here (two isolated and one joint influence).They are easy to isolate, by means of the opportunities of the new approach, having taken into consideration, that

$$\overline{m}_1 - \overline{m}_0 = \sum m_0\left(s_1 - s_0\right) + \sum s_0\left(m_1 - m_0\right) + \sum\left(m_1 - m_0\right)\left(s_1 - s_0\right),$$

[5] By means of the symbols I, II, III etc. we indicate also the following subgrowths (individual, pure, isolated and joint), and by index 0, resp. 1 – belonging to the basic period or the survey period.

where by analogy $s_1 = \frac{S_1}{\sum S_1}$, and $\sum s_1 = 1$. All the quantities s_1 for all ages characterize the age structure of the population for the the survey period.

We replace in the initial equation and obtain:

$$(\bar{m}_1 - \bar{m}_0)\sum S_0 = \sum m_0 (s_1 - s_0)\sum S_0 +$$
$$+\sum s_0 (m_1 - m_0)\sum S_0 + \sum (m_1 - m_0)(s_1 - s_0)\sum S_0.$$

The resulting three subgrowths mean:

$II_0 = \sum S_0 \sum (s_1 - s_0) m_0$ - assessment of increase in the number of deaths, caused only by the changes in the age structure of the population. It is determined as a *pure* (independent) impact by the structural factor on the total number of inhabitants and the ASMR during the basic period.

$III_0 = \sum S_0 \sum (m_1 - m_0) s_0$ - is defined as increase in the number of deaths, resulting only from the changes in the ASMR. This is the *pure* impact of the intensive factor on the number of inhabitants and the age structure during the basic period.

$II_0 III_0 = \sum S_0 \sum (m_1 - m_0)(s_1 - s_0)$ - increase in the number of deaths, caused by the *joint* changes in the ASMR and the age structure for the number of inhabitants during the basic period.

c) The third component $(\bar{m}_1 - \bar{m}_0)\sum (S_1 - S_0)$ also differs because of being complex. Following the replacements made with the value obtained in point “b” it is decomposed into three constituents:

$$(\bar{m}_1 - \bar{m}_0)\sum (S_1 - S_0) = \left(\sum S_1 - \sum S_0\right)\sum m_0 (s_1 - s_0) +$$
$$+\left(\sum S_1 - \sum S_0\right)\sum s_0 (m_1 - m_0) + \left(\sum S_1 - \sum S_0\right)\sum (m_1 - m_0)(s_1 - s_0)$$

The first expression after the equation is $I_0 II_0 = \left(\sum S_1 - \sum S_0\right)\sum (s_1 - s_0) m_0$. This is the share of the total increase in the deaths, due to the *joint* impact by the changes in the total number of inhabitants and the age structure under the basic ASMR.

The second expression is $I_0 III_0 = \left(\sum S_1 - \sum S_0\right)\sum (m_1 - m_0) s_0$. It corresponds to the share of the total growth, due to the *joint* change in the total number of inhabitants and the ASMR under the age structure during the basic period.

The third and last expression of the complete decomposition is:

$$I_0 II_0 III_0 = \left(\sum S_1 - \sum S_0\right)\sum (m_1 - m_0)(s_1 - s_0).$$

It measures the remaining share of the total growth, caused by the *joint* impact by the three factors: the total number of inhabitants, the ASMR and the age structure.

The consequence of the decomposition and subordination between growths is shown in the Scheme.

Scheme

$$(M_1 - M_0) \to \sum (S_1 - S_0)\bar{m}_0 = I_0$$

$$(M_1 - M_0) \to (\bar{m}_1 - \bar{m}_0)\sum S_0 \to \begin{cases} \sum S_0 \sum (s_1 - s_0) m_0 = II_0 \\ \sum S_0 \sum (m_1 - m_0) s_0 = III_0 \\ \sum S_0 \sum (m_1 - m_0)(s_1 - s_0) = II_0 III_0 \end{cases}$$

$$(M_1 - M_0) \to (\bar{m}_1 - \bar{m}_0)\sum (S_1 - S_0) \to \begin{cases} \left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0) m_0 = I_0 II_0 \\ \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0) s_0 = I_0 III_0 \\ \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0)(s_1 - s_0) = I_0 II_0 III_0 \end{cases}$$

Finally we established, that the total increase in the deaths for the period between the basic and the survey period is decomposed into the following seven constituents:

$$M_1 - M_0 = I_0 + II_0 + III_0 + I_0 II_0 + I_0 III_0 + II_0 III_0 + I_0 II_0 III_0 .$$

The first three subgrowths are accepted as a product of the *pure (independent)* impact by each of the three factors. The next three growths are considered the result of their *joint* combined impact ("second class" combinations), and the last – a combination of all the three factors.

The special interest in the structural influences suggests a supplementary interpretation. As evident, the structural component is included in the expressions II_0 (independent, pure effect) and in the combined effects $I_0 II_0, II_0 III_0$ and $I_0 II_0 III_0$. The following possible combinations of structural influences could be made out of these 4 key structural components. There are 6 "second class" combinations: a) $II_0 + I_0 II_0$; b) $II_0 + II_0 III_0$; c) $II_0 + I_0 II_0 III_0$; d) $I_0 II_0 + II_0 III_0$; e) $I_0 II_0 + I_0 II_0 III_0$ and f) $II_0 III_0 + I_0 II_0 III_0$. The "third class" combinations are 4 types: a) $II_0 + I_0 II_0 + II_0 III_0$; b) $II_0 + II_0 III_0 + I_0 II_0 III_0$; c) $I_0 II_0 + II_0 III_0 + I_0 II_0 III_0$ and d) $II_0 + I_0 II_0 + I_0 II_0 III_0$. The last combination of the "fourth class" consists of the above-mentioned 4 key structural components: $II_0 + I_0 II_0 + II_0 III_0 + I_0 II_0 III_0$.

As it seems however, despite the identified strict correspondences and equations the considered growths hardly reflect most precisely the result of the complicated action and interaction of these factors in reality. And yet despite the accepted conventions, the derived final equation gives an acceptable assessment of the size and nature of the factor influences. It does not refer only to the proposed scheme of decomposition. It is true in general for the schemes of the index factor analysis, as well as for some of the other measurement units of complex interactions.

2. Additive analysis at survey weights

The alternative decomposition of the total growth, but under the weights from the survey period begins with the main equation:

$$M_1 - M_0 = \bar{m}_1 \sum S_1 - \bar{m}_0 \sum S_0 = \sum S_1 \sum m_1 s_1 - \sum S_0 \sum m_0 s_0 .$$

In order to apply survey weights, we apply $S_0 = S_1 - \Delta_s$ and $m_0 = m_1 - \Delta_m$ and continue:

$$\sum S_1 \sum m_1 s_1 - \left(\sum S_1 - \sum \Delta_s\right) \sum \left(m_1 - \Delta_m\right)\left(s_1 - \Delta_s\right) =$$

$$\sum S_1 \sum m_1 s_1 - \left(\sum S_1 - \sum \Delta_s\right)\left(\sum m_1 s_1 - \sum m_1 \Delta_s - \sum \Delta_m s_1 + \sum \Delta_m \Delta_s\right).$$

And finally the total growth is decomposed into:

$$M_1 - M_0 = \left(\sum S_1 - \sum S_0\right) \sum m_1 s_1 + \sum S_1 \sum m_1 \left(s_1 - s_0\right) + \sum S_1 \sum \left(m_1 - m_0\right) s_1 -$$

$$-\left(\sum S_1 - \sum S_0\right) \sum m_1 \left(s_1 - s_0\right) - \left(\sum S_1 - \sum S_0\right) \sum \left(m_1 - m_0\right) s_1 -$$

$$-\sum S_1 \sum \left(m_1 - m_0\right)\left(s_1 - s_0\right) + \left(\sum S_1 - \sum S_0\right) \sum \left(m_1 - m_0\right)\left(s_1 - s_0\right) .$$

The components of the total growth at survey weights are interpreted analogically to the components at basic weights. The difference is only in the measurement base. Here the parameters are from the survey period.

The first component is $I_1 = \left(\sum S_1 - \sum S_0\right) \sum m_1 s_1$. It determines the share of the total growth, resulting only from the changes in the total number of inhabitants at the survey mortality (a combination of the ASMR and the structure during the survey period).

The next component $II_1 = \sum S_1 \sum \left(s_1 - s_0\right) m_1$ is the share of the total increase, resulting only from the changes in the age structure at a number of inhabitants from the survey period and the ASMR during this period.

The third expression $III_1 = \sum S_1 \sum \left(m_1 - m_0\right) s_1$ shows the size of the subgrowth, resulting *only* from the changes in the ASMR at a number of inhabitants and the age structure during the survey period.

The next expression $I_1II_1 = -\left(\sum S_1 - \sum S_0\right)\sum\left(s_1 - s_0\right)m_1$ corresponds to the share of the total growth, due to the *joint* effect from the changes in the total number of inhabitants and the changes in the age structure at the ASMR during the survey period. This component of the total growth is included with a negative sign.

The fifth share of the growth is $I_1III_1 = -\left(\sum S_1 - \sum S_0\right)\sum\left(m_1 - m_0\right)s_1$. It is due to *the joint* changes in the total number of inhabitants and the ASMR at the age structure during the survey period. This component is also included with a negative sign.

The sixth share of the total growth is determined by the expression $II_1III_1 = -\left[\sum S_1 \sum\left(m_1 - m_0\right)\left(s_1 - s_0\right)\right]$. The subgrowth is the result of the *joint* changes in the ASMR and the age structure of the population at a number of inhabitants during the survey period. This share of growth is also included with a negative sign.

The last (seventh) share of the total increase in deaths $I_1II_1III_1 = \left(\sum S_1 - \sum S_0\right)\sum\left(m_1 - m_0\right)\left(s_1 - s_0\right)$ is due to the *joint* impact from the changes in the three factors: the total number of inhabitants, the ASMR and the age structure. This subgrowth is not changed in size under the two weighting methods: $I_0II_0III_0 = I_1II_1III_1$.

It turns out, that at survey weights the total growth is decomposed in the respective seven ultimate subgrowths:

$$M_1 - M_0 = I_1 + II_1 + III_1 - I_1II_1 - I_1III_1 - II_1III_1 + I_1II_1III_1.$$

In order to be correct the above mentioned theoretical examples should be supported by practical application of the method. Therefore we test the factor contingency of the changes in the number of deaths in Bulgaria in 2000 and 2007. For convenience we compile working Table 1.

Table 1

Age (years)	Deaths		Inhabitants (thousand)		Mortality		Structure	
	2000 M_0	2007 M_1	2000 $\overline{S}_0$	2007 $\overline{S}_1$	2000 m_0	2007 m_1	2000 s_0	2007 s_1
0-14	1433	1005	1284	1028	0.001116	0.000978	0.1572	0.1342
15-29	1347	1192	1783	1573	0.000755	0.000758	0.2182	0.2054
30-44	3699	2968	1673	1636	0.002211	0.001814	0.2048	0.2136
45-59	13950	13771	1654	1628	0.008434	0.008459	0.2024	0.2125
60 +	94658	94068	1776	1795	0.053300	0.052410	0.2174	0.2343
Total	115087	113004	8170	7660	0.014086	0.014752	1.0000	1.0000

Table 1, cont.

Age (years)	Differences (changes)		Mortality ($\sum m_0 s_0$ and $\sum m_1 s_1$)	
	$(m_1 - m_0)$	$(s_1 - s_0)$	$m_0 s_0$	$m_1 s_1$
0-14	-0.000138	-0.0230	0.0001754	0.0001312
15-29	0.000003	-0.0128	0.0001647	0.0001556
30-44	-0.000400	0.0088	0.0004528	0.0003874
45-59	0.000025	0.0101	0.0017070	0.0017975
60 +	-0.000890	0.0169	0.0115874	0.0122796
Total	X	0	0.0140873	0.0147513

Table 1, cont.

Age (years)	Factor impacts				
	Basic weights		Survey weights		Neutral
	$m_0(s_1 - s_0)$	$s_0(m_1 - m_0)$	$m_1(s_1 - s_0)$	$s_1(m_1 - m_0)$	$(s_1 - s_0)(m_1 - m_0)$
0-14	-0.0000256	-0.0000216	-0.0000224	-0.0000185	0.0000031
15-29	-0.0000096	0.0000006	-0.0000097	0.0000006	0.0000000
30-44	0.0000194	-0.0000819	0.0000159	-0.0000854	-0.0000035
45-59	0.0000851	0.0000050	0.0000854	0.0000053	0.0000002
60 +	0.0009007	-0.0001934	0.0008857	-0.0002085	-0.0000150
Total	0.0009700	-.0002913	0.0009549	-0.0003065	-0.0000152

The table shows, that the deaths in 2007 are 2083 persons less compared to the basic year 2000: $M_1 - M_0 = 113004 - 115087 = -2083$.

The analysis shows, that at basic weights the decrease is a result of the following factor impacts:

a) $I_0 = \left(\sum S_1 - \sum S_0\right) \sum m_0 s_0 = -510000 \times 0,0140873 = -7184$ - negative growth in the number of deaths due to *independent* impact in the changes in the total number of inhabitants during the period (impact by the extensive factor).

b) $II_0 = \sum S_0 \sum \left(s_1 - s_0\right) m_0 = 8170000 \times 0,000970 = 7925$ - positive growth, caused by the *independent* impact from the changes in the age structure of the inhabitants (structural factor).

c) $III_0 = \sum S_0 \sum \left(m_1 - m_0\right) s_0 = 8170000 \times \left(-0,0002913\right) = -2380$ - negative growth, caused by the *independent* impact from the changes in the ASMR (intensive factor).

d) $I_0II_0 = \left(\sum S_1 - \sum S_0\right)\sum\left(s_1 - s_0\right)m_0 \quad = -510000 \times 0,000970 = -495$ - negative growth, resulting from the *joint* impact from the changes in the total number of inhabitants and the changes in the age structure.

e) $I_0III_0 = \left(\sum S_1 - \sum S_0\right)\sum\left(m_1 - m_0\right)s_0 = -510000 \times \left(-0,0002913\right) = 148$ - positive growth, due to the *joint* impact from the changes in the number of inhabitants and the changes in the ASMR.

f) $II_0III_0 = \sum S_0 \sum\left(s_1 - s_0\right)\left(m_1 - m_0\right) = 8170000 \times \left(-0,0000152\right) = -124$ - negative growth, caused by the *joint* impact from the changes in the age structure and the ASMR.

g) $I_0II_0III_0 = \left(\sum S_1 - \sum S_0\right)\sum\left(s_1 - s_0\right)\left(m_1 - m_0\right) \quad = -510000 \times \left(-0,0000152\right) = 8$ - the last share in the total growth, due to the *joint* impact from the three factors: inhabitants, ASMR and age structure.

According to the total scheme of decomposition at basic weights the total growth is a sum of the established subgrowths:

$$M_1 - M_0 = -7184 + 7925 - 2380 - 495 + 148 - 124 + 8 = -2102 \approx -2083.$$

The approximate equation is due to the rounding in the calculation procedures.

Therefore, the predominant part of the total growth is a result of the independent impact from the extensive, the intensive and the structural factor.

The decomposition at survey weights results in analogical subgrowths:

a) $I_1 = \left(\sum S_1 - \sum S_0\right)\sum m_1 s_1 = -510000 \times 0,0147513 = -7523$ - negative growth, due to the *independent* impact in the changes in the total number of inhabitants (impact by the extensive factor).

b) $II_1 = \sum S_1 \sum\left(s_1 - s_0\right)m_1 = 7660000 \times 0,0009549 = 7314$ - positive growth, caused by the *independent* impact from the changes in the age structure (structural factor).

c) $III_1 = \sum S_1 \sum\left(m_1 - m_0\right)s_1 = 7660000 \times \left(-0,0003065\right) = -2348$ - negative growth, caused by the *independent* impact from the changes in the ASMR (intensive factor).

d) $I_1II_1 = \left(\sum S_1 - \sum S_0\right)\sum\left(s_1 - s_0\right)m_1 = -510000 \times 0,0009549 = -487$ - negative growth, caused by the *joint* impact from the changes in the number of inhabitants and the age structure.

e) $I_1III_1 = \left(\sum S_1 - \sum S_0\right)\sum\left(m_1 - m_0\right)s_1 = -510000 \times \left(-0,0003065\right) = 156 -$ positive growth, due to the *joint* impact of the changes in the total number of inhabitants and the ASMR.

f) $II_1III_1 = \sum S_1 \sum (m_1 - m_0)(s_1 - s_0) = 7660000 \times (-0,0000152) = -116$ - negative share in the total growth, due to the *joint* impact from the changes in the ASMR and the age structure.

g) $I_1II_1III_1 = \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0)(s_1 - s_0) = -510000 \times (-0,0000152) = 8$ - the remaining share in the total growth, as a result of the *joint* impact of the three factors: the number of inhabitants, the ASMR and the age structure.

We have established, that at the total balanced equation the subgrowths under points "d", "e" and "f", are included with a negative sign:

$$M_1 - M_0 = -7523 + 7314 - 2348 - (-487) - 156 - (-116) + 8 = -2102 \approx -2083.$$

And at survey weights the direction of the factor impacts has not changed. However the numeric meanings of the subgrowths are different to a certain degree from those at basic weights. The extent of the differences is evident from Table 2.

Table 2

Weights	Factor components							
	I	II	III	I, II	I, III	II, III	I, II, III	Abs. sum
Survey	-7523	7314	-2348	-487	156	-116	8	17952
Basic	-7184	7925	-2280	-495	148	-124	8	18264
Abs. difference	339	611	32	8	8	8	0	312

Table 2 shows, that in both ways of weighing, the growths from the independent impact by the factors I, II and III play a decisive role. Their share compared to the total absolute sum of the growths is 95.75% at the basic weight and approximately the same at survey weights – 95.72%.

We should not comment the choice of weights here, as this is a problem complicated enough, requiring an independent and profound investigation, supported by a thorough argumentation. We shall just point out, that each decomposition scheme has its own cognitive value, and the meanings of growth at the basic weight and the survey weight could be considered limit values of the real factor impacts, and the exact meaning is difficult to be established. Due the importance of this problem however we shall try to interpret the origin of these differences, as detailed as possible, of course.

3. Analysis of the growths at basic weights and at survey weights

The complicated nature of the differences between the growth in both types of weighting could be elucidated after analyzing the absolute and relative differences.

a) The first absolute difference is:

$$I_1 - I_0 = \left(\sum S_1 - \sum S_0\right)\bar{m}_1 - \left(\sum S_1 - \sum S_0\right)\bar{m}_0 = \left(\sum S_1 - \sum S_0\right)\left(\bar{m}_1 - \bar{m}_0\right) = 339.$$

It is due to the difference in the total number of inhabitants and the difference in the total (mean) mortality during the basic and the survey period.

It turns out, that the relative difference between both types of growth is mainly due to the difference in the total mortality and the contingent factors (the ASMR and the age structure).

$$\frac{I_1}{I_0} = \frac{\bar{m}_1}{\bar{m}_0} = \frac{\sum m_1 s_1}{\sum m_0 s_0} = 1,047 .$$

b) The second absolute difference is:

$$II_1 - II_0 = \sum S_1 \sum (s_1 - s_0) m_1 - \sum S_0 \sum (s_1 - s_0) m_0 = 611 .$$

It is due to the difference in the total number of inhabitants during the two periods and in the structural changes, measured respectively compared to the basic and the survey ASMR.

The relative difference in this case is determined by the change in the total number of inhabitants, as well as in the structural changes, relative to the changes in the ASMR:

$$\frac{II_1}{II_0} = \frac{\sum S_1}{\sum S_0} \times \frac{\sum (s_1 - s_0) m_1}{\sum (s_1 - s_0) m_0} = \frac{\sum S_1}{\sum S_0} \times \frac{\left(\bar{m}_1 - \bar{m}_1^{(s_0)}\right)}{\left(\bar{m}_0^{(s_1)} - \bar{m}_0\right)} = 0,93757 \times 0,9844 = 0,9229.$$

c) The third absolute difference is defined by the expression:

$$III_1 - III_0 = \sum S_1 \sum (m_1 - m_0) s_1 - \sum S_0 \sum (m_1 - m_0) s_0 = 32.$$

It is determined by the difference between the total number of inhabitants during the two periods and the change in the ASMR, measured respectively compared to the basic and survey age structure.

The relative difference depends on the changes in the number of inhabitants and on the changes in the ASMR, relative to the structural changes:

$$\frac{III_1}{III_0} = \frac{\sum S_1}{\sum S_0} \times \frac{\sum (m_1 - m_0) s_1}{\sum (m_1 - m_0) s_0} = \frac{\sum S_1}{\sum S_0} \times \frac{\left(\bar{m}_1 - \bar{m}_0^{(s_1)}\right)}{\left(\bar{m}_1^{(s_0)} - \bar{m}_0\right)} = 0,93757 \times 1,05218 = 0,98649.$$

d) The fourth absolute difference is:

$$I_1 II_1 - I_0 II_0 = \left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0) m_1 - \left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0) m_0 = 8.$$

It depends on the difference in the number of inhabitants during the basic and the survey period, as well as on the changes in the structure, measured in comparison to those of the ASMR.

However regarding the ratio between both types of growth there is no impact by the extensive factor (the inhabitants). Only the effect by the structural changes is maintained, in view of the changes in the ASMR:

$$\frac{I_1 II_1}{I_0 II_0} = \frac{\sum (s_1 - s_0) m_1}{\sum (s_1 - s_0) m_0} = \frac{\left(\overline{m}_1 - \overline{m}_1^{(s_0)}\right)}{\left(\overline{m}_1^{(s_1)} - \overline{m}_0\right)} = 0,9844.$$

It should be pointed out, that this ratio is a component of the ratio $\frac{II_1}{II_0}$.

e) The absolute difference in the fifth subgrowth is:

$$I_1 III_1 - I_0 III_0 = \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0) s_1 - \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0) s_0 = 8.$$

The difference is due to the changes in the number of inhabitants and in the ASMR, measured against the structural changes.

Also for this relative difference there is no impact by the extensive factor:

$$\frac{I_1 III_1}{I_0 III_0} = \frac{\sum (m_1 - m_0) s_1}{\sum (m_1 - m_0) s_0} = \frac{\left(\overline{m}_1 - m_0^{(s_1)}\right)}{\left(\overline{m}_1^{(s_0)} - \overline{m}_0\right)} = 1,05218.$$

It is not difficult to note, that this ratio is a component of $\frac{III_1}{III_0}$.

f) The sixth absolute difference is derived by the expression:

$$II_1 III_1 - II_0 III_0 = \sum S_1 \sum (m_1 - m_0)(s_1 - s_0) - \sum S_0 \sum (m_1 - m_0)(s_1 - s_0) = 8.$$

It is due to the changes in the total number of inhabitants, combined with the changes in the ASMR and the age structure.

The relative difference in this case is determined only by the changes in the total number of inhabitants:

$$\frac{II_1 III_1}{II_0 III_0} = \frac{\sum S_1 \sum (m_1 - m_0)(s_1 - s_0)}{\sum S_0 \sum (m_1 - m_0)(s_1 - s_0)} = \frac{\sum S_1}{\sum S_0} = 0,93757.$$

g) In the last component of the total growth there is no difference between the two types of weighing:

$$I_0 II_0 III_0 = I_1 II_1 III_1 = \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0)(s_1 - s_0) = 8.$$

In addition to the established relations between the combined impacts of the factors weighted in a different way other interesting facts are available as well. For instance:

$$\left(I_1 II_1 - I_0 II_0\right) = \left(I_1 III_1 - I_0 III_0\right) = \left(II_1 III_1 - II_0 III_0\right) = I_0 II_0 III_0 = I_1 II_1 III_1 =$$
$$= \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0)(s_1 - s_0) = 8.$$

The above shows, that the differences between the growths with combined impact at the basic weight and the survey weight are equal to each other. This equation also shows precisely, that they are equal to the joint impact of the three factors, and hence the conclusion, that this impact is decisive also for the formation of the differences.

Other links are available as well, to reveal and supplement the main part of the analysis. For example the unification of the structural subgrowths II_0 and II_0III_0 (the “second class” combination according to point “b”) results in a new type of structural impact, combining the total volume of the extensive factor during the basic period with the group intensities during the survey period:

$$II_0 + II_0III_0 = \sum S_0 \sum (s_1 - s_0)m_0 + \sum S_0 \sum (m_1 - m_0)(s_1 - s_0) =$$
$$= \sum S_0 \sum (s_1 - s_0)m_1.$$

Interesting result is obtained also with the combination:

$$II_0 + I_0II_0 = \sum S_0 \sum (s_1 - s_0)m_0 + \left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0)m_0 =$$
$$\sum S_1 \sum (s_1 - s_0)m_0.$$

This is the structural “second class” combination according to point “a”.

The final combination as per point “e” is not less interesting:

$$I_0II_0 + I_0II_0III_0 = \left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0)m_0 + \left(\sum S_1 - \sum S_0\right) \sum (m_1 - m_0)(s_1 - s_0) =$$
$$\left(\sum S_1 - \sum S_0\right) \sum (s_1 - s_0)m_1 = I_1II_1.$$

The result of the transformation is amazing – a joint structural impact was obtained, entirely made of survey weights.

These few examples are an evidence for the available strict links between the growths at the basic weights and at the survey weights. This is of paramount importance for the proper set-up of the logical schemes of the index factor analysis and for increasing their cognitive opportunities.

Other matches and links could be pointed out as well. For instance the cognitive meaning of the ratio $\frac{I_1}{I_0}$ is revealed much more completely, when looking for its link to the common index of structure (1.0670) and the index of mortality (0.9787).[6] In this case the analysis is enriched with the appearing multiplication link:

[6] The evaluation of the size of structural changes was obtained from the “total index of the structure”, suggested in the demographic statistics:

$$\frac{I_1}{I_0} = 1,0670 \times 0,9787 = 1,044 \approx 1,047\ .$$

It is also established, that:

$$\frac{II_1}{II_0} = \frac{II_1 III_1}{II_0 III_0} \times \frac{I_1 II_1}{I_0 II_0} = 0,93757 \times 0,9844 = 0,9229.$$

Additional information is also included in:

$$\frac{III_1}{III_0} = \frac{II_1 III_1}{II_0 III_0} \times \frac{I_1 III_1}{I_0 III_0} = 0,93757 \times 1,05218 = 0,98649.$$

It is not difficult to make the corresponding conclusions for each of the above-mentioned links. Other similar relations are available as well, which have not been studied here. Considered in their integrity and interrelation, they would be a good basis for useful reflections on the specifics of the index theory.

Finally it could be summed up, that the above-mentioned theoretical foundations of the new approach for carrying out an additive index factor analysis enable the formulation of a more strict, more thorough and more logical scheme for decomposition of the total growth. The large scale discussions made in theoretical and application aspect support the idea, that the proposed theory enlarges the opportunities of the analysis. The reflections made on the complicated nature of the components of the total growth, their origin and their factor basis supplement the theory. Some limited doubts were stated, related mainly with the usual conventions of the index analysis. We should not forget however, that the doubts and the demand in science are very useful for its development.

The author is hoping to focus the attention of our statisticians with this article. Other opinions are welcome as well.

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$I_{str} = \dfrac{\sum m_1 S_1}{\sum m_0 S_0 \dfrac{\sum m_1}{\sum m_0} \times \dfrac{\sum S_1}{\sum S_0}} = 1,0670$. “the non-weighted mortality index” was used as well:

$$I_m = \frac{\sum m_1}{5} : \frac{\sum m_0}{5} = 0,9787.$$

Its is confirmed that $I_{\bar{m}} = \dfrac{\bar{m}_1}{\bar{m}_0} = I_{str} \times I_m$, a $I_M = \dfrac{M_1}{M_0} = I_{\bar{m}} \times I_s = 0,9816 \approx 0,9819.$